
Application of Statistical Inference Using Entropy to Characterize the Transfer of Data Across Financial Systems

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Abstract: This paper explores the application of statistical inference using entropy to characterize the transfer of data across financial systems. The transfer of data between financial systems is a critical process that affects the accuracy and reliability of financial information. In this study, we propose a new approach to quantify the amount of information transferred between two financial systems based on entropy measures. Specifically, we use Transfer entropy to measure the information content of data transferred between financial systems. The Kullback-Leibler separation of conditional probabilities is a design statistic known as Transfer entropy. This method enables the determination, measurement, and testing of information transmission without being limited to linear movements. We demonstrate the effectiveness of our approach by analyzing using this empirical technique, the importance of the borrowing exchange industry in comparison to the exchanges for debt securities. In addition to this, this paper investigates the greater connectivity that exists among operational risks, as represented, respectively, by iTraxx and VIX Europe. We show that our entropy-based approach can detect changes in the information content of data transferred between the two exchanges, which can be indicative of changes in market conditions or the behavior of market participants. Overall, this study highlights the potential of entropy-based statistical inference methods for characterizing the transfer of data across financial systems. This approach has the potential to provide valuable insights into the behavior of financial markets and the efficiency of financial systems, which can inform policy decisions and improve the accuracy and reliability of financial information.

Keywords: Transfer Entropy, The Kullback-Leibler Divergence, iTraxx, VIX

1. Introduction

Discovering and quantifying connections among distinct periods is a topic of study in many fields. The informational relationship between various marketplaces is of particular significance in finance [1]. However, there are just a few approaches for experimentally investigating these connections. Granger causality is extensively used to find correlations among time-stamped data [2]. Nevertheless, the information derived through this strategy is confined to the sheer presence of different parts of the network instead of their measurement. Only for a specific set of practical applications does an exact measure for transmitting information among financial markets arise: if price increases in various markets pertain to single underlying securities,

market pricing measures, including Hasbrouck's [3] information share capital, can be utilized to ascertain the information-based dominant industry. This technique required a vector error correction correlation between the various time sequences in order to function properly. However, it really is the only one who can offer a conclusive result regarding the results if the link among the elements is proposed by theory and supported by verifiable evidence. In other words, it is the only one that can provide a logical conclusion about the outcomes. In addition to that, this technique calls for a vector error correction correlation to be established between the several time sequences. In addition, Granger causality and market pricing indicators are constructed on an access basis as a vector autoregressive model (VAR) or a vector error correction model (VECM),

both of which call for quite strong assumptions on the underlying processes that are linear.

This piece of research proposes making use of transfer entropy as an alternative method for quantifying information across various time series in finance. Transfer entropy is a model-free statistic that may be computed by determining the Kullback-Leibler difference between the likelihood function. Under inadequate conditions, this method quantifies information transmission without being limited to linear movements. It is indeed an exciting tool if the data does not match the hypothesis needed by typical models. Just a few researchers have used entropy in the setting of institutions regarding finance. Regularly, Kwon and Yang [4] examined the flow of information among the S&P 500, the indices by Dow Jones, and chosen firms. Baek et al. [5] investigated the direction and magnitude of information transmission among organizations of NYSE-listed equities in order to determine market sensitivity and the top companies. Research by Reddy and Sebastin [6] looks at the assessment of interconnections between the Indian stock and commodities markets.

Statistical inference was not taken into consideration in any of this financial research. This paper deals with this issue and suggests bootstrapping the fundamental Markov process [7] to figure out the share of predictions and evaluate how significant the predicted flow of data is statistically. Moreover, the paper uniquely approaches the problem of adjusting for limited bias with the sample in the transfer entropy appraiser. The paper also proposes using the built-from-scratch distribution with the null hypothesis that there is no transfer of information to rectify for bounded sampling error instead of randomized information, which has previously been seen as the normal practice in the study of theories regarding information thus far.

2. Using Transfer Entropy to Measure Information Dissemination

It is best to understand dissemination entropy in the context of information theories. Hartley [8] devised an informational methodology based on the logarithmic of the total number of conceivable symbol combinations during the early days of electric transfer, when Morse code was used for communications. The main objective was to encrypt messages for faster transmission. To achieve that goal, it was crucial to quantify the content obtained from detailed information on symbols. Think about the following instance: There seem to be two equally probable outcomes—head or tail—when tossing a coin. According to Hartley [8], $H = \log$ contains the data that may be obtained from a single coin flip (denoted by H). $H = \log_2(2) = 1$, and bits will serve as the measuring unit if the logarithm's baseline is 2. As a result, n coin flips yield n data instances ($H = \log_2(2^n) = n \log_2(2) = n$). Therefore, n binary digits are required to indicate the outcome (such as 1 for heads and 0 for tails).

Suppose signals are not entirely possible but occur with separate probabilities, P_j . In that case, the magnitude of the

data acquired from a distinct symbol j may be calculated as $\log(1/p_j)$, where P_j is the likelihood that the sign occurs. The formula $H = -\sum_j p_j \log_2 p_j$ can be used to calculate the average amount of data (per symbol) acquired from such a sequence. In this calculation, n refers to the total number of unique signals encountered. This is related to Shannon's general formula from 1948: take J to be a randomly distributed variable possessing the probabilistic model $p(j)$, wherein j symbolizes the four distinct measures (or states) for J . Shannon's formula for the probability density function of discrete random variables is denoted by $p(j)$. The Shannon entropy, shown in the following passage, consequently offers the mean number of bits needed for the optimal encoding of distinct choices taken from the distributions of J .

Equation 1: Shannon's formula

$$H_j = -\sum_j p(j) \times \log_2 p(j)$$

In the following, "log" will refer to the logarithm based on base 2, and the sum will be determined by adding up all of J 's different values. The equation after this paragraph depicts Shannon's formula, which measures uncertainty. The ambiguity around potential implementations of J increases as more bits are required to correctly capture the revelations of the processes. The most significant degree of uncertainty will be available if J is fairly disbanded, which means that a lottery choice will create any establishment of J having an identical chance and if all amounts of J are similarly likely. This will cause the maximum amount of uncertainty to be available. To demonstrate the connection between ambiguity and knowledge, the Kullback-Leibler separation is used [9]. When a posteriori data $q(j)$ of J that is distinct from $p(j)$ is wrongly assumed, it explains why more bits are necessary for decoding:

Equation 2: Shannon's formula

$$K_j = \sum_j p(j) \times \log \frac{p(j)}{q(j)}$$

Diagnostics and Inferences

The choice of k and l in the following equation determines the ordering of the Markov process, which determines how the transfer entropy measurements are calculated. To record the data movement between two time-stamped datasets, k and l must be large enough. Nevertheless, when the block length is increased, the impacts of the limited sample will worsen. However, if k in $T_{JI}(k, l)$ is too little, knowledge from prior I values can be mistakenly attributed to J . This will not happen if I am autonomous from myself and have a delay of k . As a result, the researchers calculate the cosine similarity in both the time-stamped data and their sequence while taking into account the latency k to determine the optimal prerequisites.

Equation 3: Procedure by Markov

$$M_{\Pi_k}(k) = \sum_{i, i_k} p(i, i_k) \times \log \frac{p(i, i_k)}{p(i)p(i_k)}$$

The k values that coincide with the initial local solution of this characteristic are employed to construct the optimum

main measures, which refer to the duration of every brick. When Ik indicates the sequence that I began but was delayed

indefinitely by k , the duration of every square is determined by the number of every block.

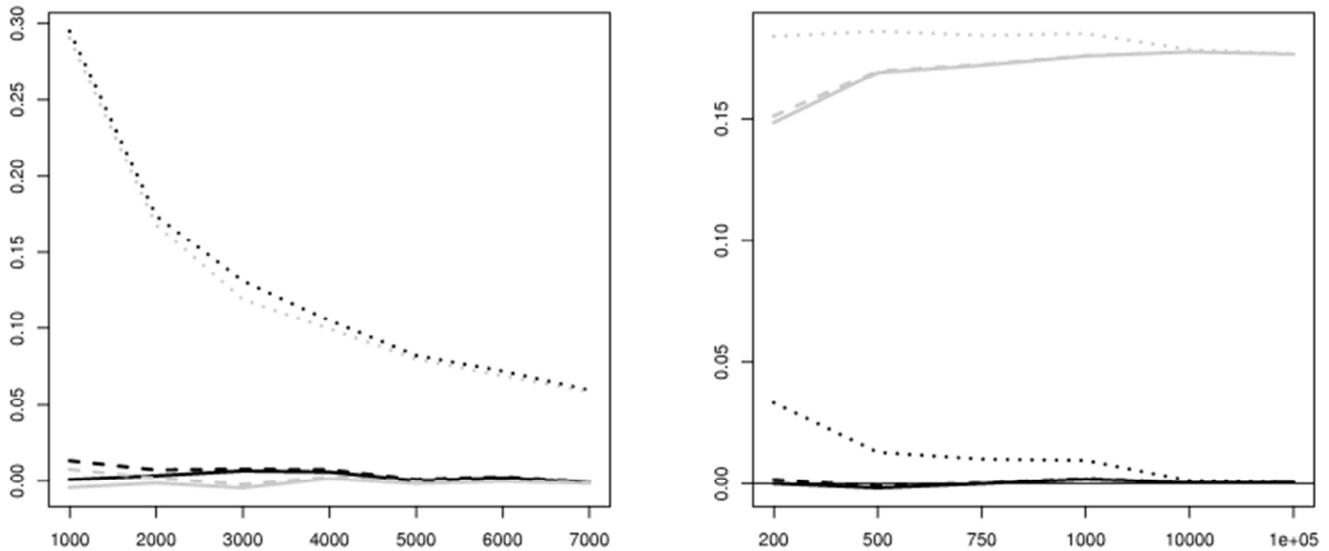


Figure 1. Example of a very tiny bias in the population.

It is critical to estimate the changeover probabilities from such a specific sample in order to construct entropy measurements. The transfer entropy predictions obtained from the equation below are expected to be skewed due to minor population effects, according to Marschinski and Kantz [10]. They propose that the institutional aspects of entropy should be changed as follows:

Equation 4: Transfer entropy predictions

$$ET_{J \rightarrow I}^{shuffled}(k, l) = T_{J \rightarrow I}(k, l) - T_{J_{shuffled} \rightarrow I}(k, l)$$

where the transfer entropy for the scrambling series may be represented by the notation $T_{J_{shuffled} \rightarrow I}$. Through randomly assigning revelations out of the population of J as well as reconfiguring them to create another time-stamped dataset, scrambling is achieved. As a result, there are no longer any statistical relationships between the two time variables. Consequently, $T_{J_{shuffled} \rightarrow I}(k, l)$ will always be equal to zero when giving participants; any value that is not zero will be due to the effects of having a smaller sample size. When calculating the entropy of contracted telecommunications companies, it is common practice to first scramble the information several times before using the transfer volatility evaluation. This is done in conjunction with bias assessment, which is determined by determining the external service providers' entropy. Data rearrangement also results in the destruction of the linkages within the multivariate time series. A setup that maintains the dynamic nature of the time series data and delivers distributions of transfer entropy estimates on the assumption that there is no flow of information enables a more exact identification of the slight sample bias. How to determine the predicted flow of information is closely tied to this problem.

3. Empirical Examples

3.1. Flows of Information Among Credit Default Swap (CDS) Market and the Commercial Credit Markets

As the disparity between hazardous government credit and rates without risks also affects the bond threats of a particular firm, the CDS structure is related to the market for corporate bonds. There is, thus, a rough arbitrage relationship between the CDS and the commercial credit market since both marketplaces value creditworthiness. This arbitrage relationship has the following explanation: A buyer purchases a T -year par bond from the reference business with a maturity rate of y . Additionally, a businessman acquires insurance for the firm through a loan from CDS. Through arbitrage and also because credit rating is avoided, the annual return, represented by y pCDS, must be equivalent to the T -periods rate without risks, written by x . Cutting the bonds with threats, buying insurance on the CDS structure, and purchasing the rates without taking risks would create opportunities for arbitrage if y pCDS $>$ x . They are devaluing the risk-free bond, and purchasing the 11 risky bonds and insurance becomes beneficial if y pCDS $>$ x . Therefore, $pCDS = pCS = yx$ should be the CDS price equal to the margin requirement. However, the arbitrage connection is not flawless due to market flaws, including liquidity premiums, imperfectly matched maturities, and the lowest ability to provide options in the event of default. The approximate character of the arbitrage link involving CDS pricing and credit spreads is considered via cointegration.

A VECM model and the Hasbrouck [3] information exchange are used in most empirical research that looks at the significance of the CDS and financial markets. Because markets are not flawless, a VECM can't be appropriate for

the given time series, and empirical data seldom supports correlations. As a consequence of this, the authors of this work propose making use of transfer entropy as an alternative, model-free technique. This methodology's benefit over Hasbrouck's [3] information sharing is that it does not need a cointegration connection. Even so, it holds true if the

data is perfectly correlated. The data must be stable to employ transfer entropy, which may be achieved by obtaining the initial variations [11, 12]. Because of this, entropy is also critical if Hasbrouck's [3] principles are broken and may be used in more situations.

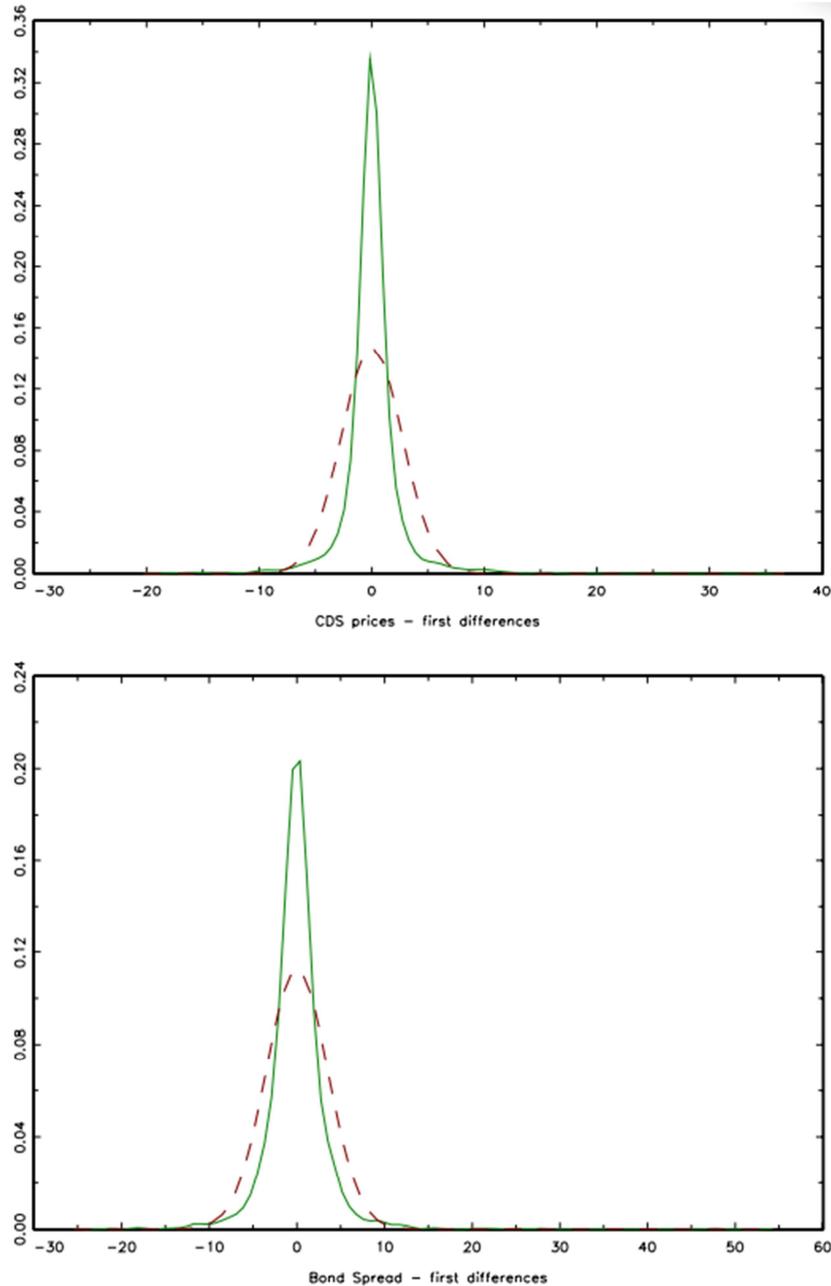


Figure 2. BASF CDS and credits gap initial discrepancies are shown as statistical charts.

A few further allusions Entities can transport entropy predictions that are statistically relevant in both directions. There is a substantial transmission of information in one direction for nine reference organizations. During a crisis, the communication connection between the two markets becomes stronger (phase 2). For the preponderance (23 out of 27) of the referenced organizations, efficient transfer entropy estimations reveal statistically meaningful bidirectional take-

up of the information. In 16 reference firms, the data transmission within the CDS market is greater than the data dissemination within the financial markets. Similar trends are evident in Phase 3; efficient transfer entropy predictions are considerable for all but one comparable firm (EDF). Within 11 corporations, the dissipation of data out of the CDS market is greater than the information flow that originated in the financial markets. Ultimately, the findings demonstrate

that data flow among the CDS as well as the credit markets grew with time, with the relevance of the CDS market peaking during the financial recession. This is consistent with findings from research by Coudert and Gex [13], indicating that the CDS market outperforms the bond market for sovereign governments, particularly during the current crisis.

3.2. Information Exchange Among Market Risk and Credit Risk

The paper has demonstrated that the CDS marketplace is crucial in the process of valuing credit risk. However, for agents to price CDS, companies need knowledge of credit risk, namely the likelihood that the fundamental reference

firm will fail. Credit agencies may provide some information. However, it is still unclear if data on credit risk may also be gleaned from the financial markets. The Merton model [14] provides a theoretical connection between credit risk and the equity markets. Studies using actual data also show this connection. Byström [15] investigates the correlation between the businesses' inventory price changes and the iTraxx indices, which serve as a gauge of credit risk. It discovers a good link between the iTraxx and equity index returns' variability. In addition, Longstaff et al. [16], Collin-Dufresne, Goldstein, and Spencer [17], and Pan and Singleton [18] all found that there is a link between governmental trustworthiness and variability indices.

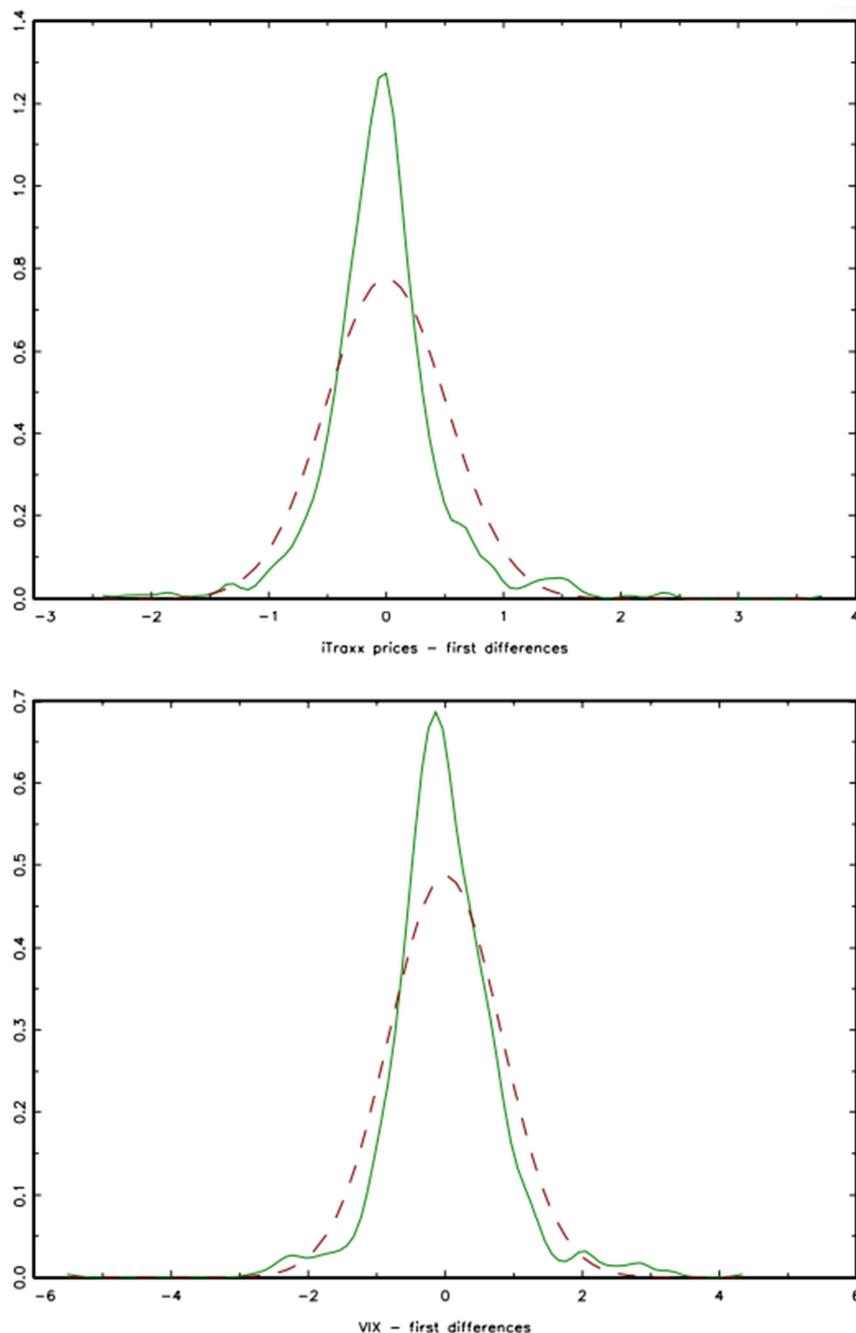


Figure 3. First discrepancies in the iTraxx and credit VIX kernel density graphs.

The findings of this paper indicate that there was information flow in both directions before the crisis. Both sides are substantial in the initial binarization, with the iTraxx overwhelming the VIX. Nevertheless, there is no statistically significant distinction. For the residual bindings, the transmission entropy way of measuring picks up far too much noise, causing the bias forecasts to keep growing faster than the appraiser for the transmission entropy itself. Inside the second binarization, the situation is reversed: the paper finds considerably higher transmission electron density forecasts for the transfer of data from VIX to the iTraxx. This results from the information-gathering strategies' recommended large number of delays. The VIX's

significance is made more apparent during the crisis since the institutional entropy from the VIX to the iTraxx is very important for three bins. Only the initial binning is relevant in the opposite direction (albeit more minor). The noisy element seems to reappear in some of the remaining bindings, and transfer entropy predictions turn negative, indicating that there is no address for a wide range from the iTraxx to the VIX and emphasizing once again the VIX's supremacy currently. Although the VIX remains dominant after the crisis, our findings imply that the market situation has returned to unidirectional information dissemination. Even if not across all binning, the paper finds a considerable ability to transport entropy predictions in both structures.

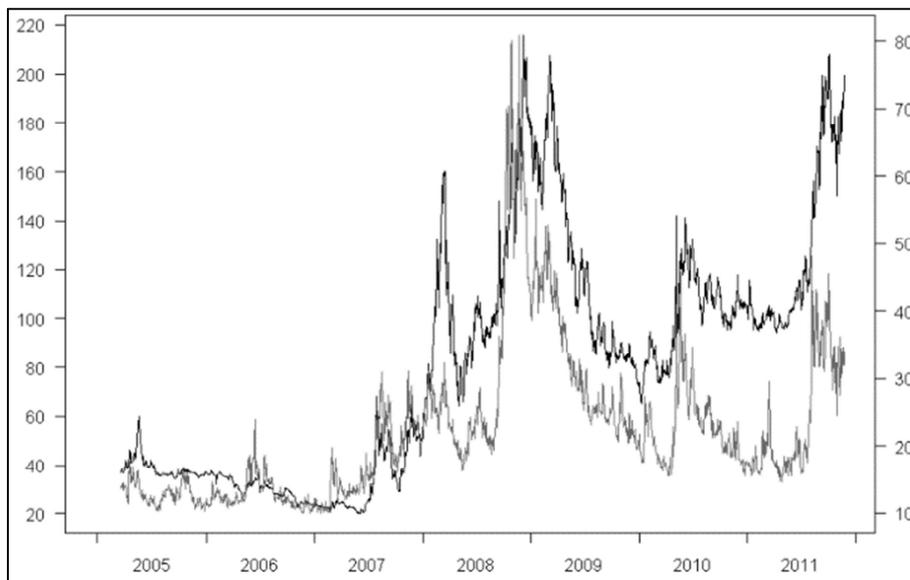


Figure 4. *iTraxx and VIX.*

The paper concludes that a changing interplay among credit risk and market risk prevails, with the VIX serving as the primary information source for information transmission to iTraxx. These two market structures are distinct; the iTraxx is sold above a white line, whereas the VIX is calculated over a set period of time. Consequently, other (macroeconomic) elements included in the VIX may be the reason for the transfer of data from the VIX to the iTraxx when the manual OTC dealing of the iTraxx may create latency in the inclusion of information. Nevertheless, this problem should have little consequence and not obscure our overall results since the research is based on daily data. This topic may be the subject of future study because, whenever measuring transfer entropy, it is often feasible to term other factors to exclude the typical response to an external factor from the following information flows. However, the number of conditional probabilities that must be calculated from the population is increased by an additional variable, necessitating more studies to reach somewhat accurate conclusions. Additionally, it seems significant and should be considered when evaluating transfer entropy estimates because the observations were divided into several bins.

4. Conclusion

The flow of information across financial markets is investigated in this research using the idea of transfer entropy. The transfer entropy may quantify the amount of data exchanged based on the Kullback-Leibler separation. Its main advantages include being non-parametric and considering both regular and non-linear motions. As a result, it offers an intriguing substitute for accepted methods like Granger causality, which could also identify information transmission but cannot evaluate it. The bootstrap method suggested in this study enables the execution of transfer entropy prediction inference, a problem that has not yet been tackled. The paper was able to determine the statistical significance of the transfer entropy predictions more precisely by bootstrapping the fundamental Markov process instead of rearranging the information or depending on block bootstrapping. The bias in the transfer entropy predictions is corrected using the same technique. It is demonstrated that this approach has better qualities than the typical shuffling process.

The information exchange approach of Hasbrouck [2], which depends on the presence of a cointegration connection, is the most commonly used methodology to evaluate data communication across financial markets. Transfer entropy provides an intriguing substitute for serial correlation in detecting and quantifying information flows. paper employed the transfer entropy notion to investigate data transmission between the CDS and financial markets. The findings demonstrate that, despite information flow in both directions, the CDS marketplace dominated the credit risk market. The significance of the CDS market peaked during the recession, as seen by our analysis of several timeframes. It rose over time as data communication between the CDS and bond markets expanded. Furthermore, the information flow was likely unidirectional before the current economic crisis. Still, at that time, the VIX's significance became apparent, providing an intriguing finding about credit risk assessment.

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